

1 SM162 Computer Calculus II

1.1 Final Examination

1330 Wednesday

May 2, 2001

1.1.1 Calculators may be used throughout but show all steps for full credit.

1. Find $f'(x)$ for $f(x)$ given by:

(a) $\ln(\cos^2(2x))$

(b) $\arcsin(x^2)$

Find $\int f(x) dx$ for $f(x)$ given by:

(a) $\frac{1}{x(x-1)}$

(b) $\frac{x^2}{1+4x^2}$

2. Show that the function given by $f(x) = 1 + 2x + x^3$ has an inverse. Find the equation of the tangent *to the inverse function* at $(4, 1)$.

3. Find the centroid of the region enclosed by the curves $y = 2x^2 + 1$ and the line $y = 3$.

4. A 500 lb load of bricks is attached to a cable which weighs 2 lb per foot. Find the amount of work done in lifting the load to the roof of a building 200 feet tall.

5. A bacteria culture has an initial population of 400. After one hour the population has grown to 600. What will the population be after two hours? When will it reach 1200?

6. The current in an electrical circuit containing one resistor and one inductor is given by

$$L \frac{dI}{dt} + RI = E(t)$$

where $E(t)$ is the supplied voltage. Suppose that the resistance is 15Ω , the inductance is 5H and the voltage supply is a 60V battery. The switch is turned on at $t = 0$. Find $I(t)$. Also find the limiting (steady state) voltage as $t \rightarrow \infty$.

7. **a.** Evaluate the limit $\lim_{x \rightarrow 0} \frac{2x - \sin 2x}{x^3}$.

b. Use the Comparison Test to show that the integral $\int_e^\infty \frac{dx}{(\ln x) \sqrt{x^3 + 5x - 1}}$ is convergent.

8. For each of the following series, determine whether it is *absolutely convergent*, *conditionally convergent* or *divergent*.

(a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2n^2 + 1}$

(b) $\sum_{n=1}^{\infty} \frac{(-3)^n}{2^{2n} + 1}$

(c) $\sum_{n=0}^{\infty} \frac{2^n n!}{(n+2)!}$

9. **a.** Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{3^{n-1} x^n}{n^2}$

b. It is known that for $|x| < 1$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 \dots$$

- i.** Use this information, and integration, to obtain the series representation

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \dots = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

- ii. Use the series in (i) to evaluate $\ln(1.2)$ to 3 decimals.
10. **a.** Sketch the curve whose parametric equation is $x = \cos 2t$, $y = \sin t$. Find the slope of this curve where $t = \pi/4$.
- b.** Write down **but do not evaluate** the integral for the arclength of the curve given in part (a) between $t = 0$ and $t = \pi/2$.
- c.** Find the area enclosed by one loop of the curve $r = \cos 3\theta$.